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Heat Transfer in a Homogeneous Suspension Including Radiation and History Effects

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An analytical formulation of the thermal response of particles subjected to time-dependent temperature perturbations in the surrounding medium is presented. The suspension of particles is considered homogeneous and dilute. The continuous medium containing the dispersion of particles is assumed to be weakly participating, having a small but nonzero absorption coefficient. The particles in suspension are small, so that the mechanisms of heat transfer between the particles and the continuous phase are conduction and radiation only. The general solution for the temperature response of the particles to time-dependent perturbations in the continuous phase is derived for the limit of small Biot and infinitesimal Péclet numbers. The method used to derive the general solution consists of including the linearized radiation effects in the integro-differential equation that describes the temperature history of the particles. A fractional-differential operator that contains a Riemann–Liouville–Weyl half-derivative term is then applied to the radiation-diffusion equation to render the governing equation analytically tractable. The resulting equation is solved exactly by the method of variation of parameters. Linear and harmonic perturbations are analyzed and discussed, and the radiative and history term contributions to the temperature response of the particles are studied.

Nomenclature

A_{sp} = superficial area of the particles
a = radius of particle
B = blackbody radiosity
Bi = Biot number, ha/κ_p
C_R = dimensionless radiation coefficient, Eq. (22)
c = specific heat capacity
D = steady-state conduction term coefficient, Eq. (19)
H = history term coefficient, Eq. (20)
h = heat transfer coefficient
I = intensity of radiation
k_{am} = total absorption coefficient
L_{mbg} = mean geometrical beam length
N_p = particle number density
Pe = Péclet number, $RePr$
Pr = Prandtl number, ν_m/α_m
p_i = internal optical property, Eq. (11)
Q = heat transfer rate
q = heat flux or volumetric heat source
Re = Reynolds number, $a|\boldsymbol{V}_p - \boldsymbol{V}_m|/\nu_m$
r = radial coordinate measured from the center of the particle
T = temperature
$\langle T \rangle$ = mean temperature level, $[\tilde{T}_m(0) + T_p(0)]/2$
t = time
U = unsteady forcing term coefficient, Eq. (21)
V = volume, or velocity vector at the particle position
α = parameter defined in Eq. (30), thermal diffusivity
β = parameter defined in Eq. (31)
ε = parameter in the linear forcing term
θ = temperature potential, $[T_p(\hat{t}) - \tilde{T}_m(\hat{t})]/\langle T \rangle$
κ_m = thermal conductivity of the medium
λ = heat capacity ratio, $\rho_m c_m/\rho_p c_p$
ν = kinematic viscosity of the medium
ξ = dimensionless amplitude of the harmonic perturbation
σ = Stefan–Boltzmann constant
τ = transmissivity in Eq. (10) or dummy variable of integration in Eq. (27) et seq.
ω = dimensionless frequency in the unsteady forcing term
ϵ = emissivity

Subscripts
H = related to harmonic perturbation
i = related to internal optical property
in = incoming
L = related to linear perturbation
m = related to the medium
p = related to the particles
r = at position r
ν = at a wave number ν

Superscripts
V = volumetric
$\hat{}$ = dimensionless
$\sim$ = related to the medium in the absence of particles

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I. Introduction

SEVERAL industrial processes utilize dilute, dispersed multiphase environments to either promote or take advantage of the heat transfer exchange between the phases present. In other processes the interaction is passive, where the heat transfer is just inherent to the presence of the different phases, and may not be desirable. Dilute, dispersed multiphase environments of relevance include sprayed and pulverized fuel flames, seed dryers, the pneumatic transport of particulate and powders, dilute separators, and many other processes occurring in high- and low-temperature equipment. In atmospheric sciences, attention is given to the dispersion of small particles and their thermal behavior when experiencing changes in the thermodynamic state of the different layers of gases in the atmosphere. The understanding of the thermal interaction between the phases in these processes is important to determine other related phenomena such as initial drying rates, the onset of homogeneous and/or heterogeneous chemical reactions, or thermal wearing of the dispersed material. This work provides a theoretical approach to the radiative and diffusive heat transfer problem between the continuous and the dispersed phase in a dilute, homogeneous cloud of particles in suspension.

The transient heat transfer from a spherical particle to a time-dependent surrounding temperature field is modified when compared to the quasisteady formulation of the problem. The heat transfer enhancement or diminishment is a result of the developing temperature profile around the particle.[1] This contribution is usually neglected in heat transfer analyses because it is commonplace to assume an average heat transfer coefficient that is independent of time. The contribution from the unsteadiness of the temperature profile in the near region of the particle is, however, not always negligible and can account for significant modification of the quasisteady temperature behavior.[2,3] Transient effects on the heat and mass transfer rate from a sphere have been studied numerically,[4,5] and the effect of radiation heat transfer on a single droplet heating and vaporization has been considered,[6] but few attempts to analytically treat the problem were carried out.[3] A simplified analytical model describing the transient diffusion effects in nondilute suspensions is discussed in Ref. 7. One of the objectives of the present study is to determine the conditions for which the transient effects become important, as well as to assess the effect of the radiation heat transfer mode on the temperature history of particles in a dilute suspension.

The problem studied here is that of a suspension of small particles that are either stationary or moving with small relative velocity with respect to the surrounding medium (small particle Reynolds number), so that the Péclet number of the suspension is also small. The volumetric density of particles is low, implying that the distance between particles is much larger than the radius of the particles (volumetric densities of the order of 10^{-4} or lower). Transient radiation and diffusion heat transfer between the phases is studied. Linear and harmonic perturbations on the initial state of thermal equilibrium between the phases are analyzed, and the response of the dispersed phase is discussed.

Section II presents the diffusion heat transfer problem. Section III discusses approximations for the radiation heat transfer process and restricts the absorption coefficient of the continuous medium for which the theory applies. In Sect. IV, the coupling between the diffusion and the radiation modes is discussed, and a dimensionless coefficient that quantifies the contribution of the radiation mode is identified. In this same section, the radiation–diffusion equation that describes the phenomena under study is presented. In Sec. V, the formal analytical solution of the radiation–diffusion equation is derived. Sections VI and VII are devoted to the analysis of two different kinds of perturbations on the surrounding temperature and the influence of those perturbations on the temperature of the dispersed phase.

II. Diffusion Problem

Consider unsteady diffusion heat transfer in a medium surrounding a spherical particle of radius a. Let m stand for properties of the medium and p for properties of the particles. The one-dimensional, time-dependent temperature field around the sphere is governed by the solution of the following transport equation:

$$\rho_m c_m \frac{\partial T_m}{\partial t} = \frac{\partial}{r^2 \partial r}\left(r^2 \kappa_m \frac{\partial T_m}{\partial r}\right) + q_r^V, \qquad r > a \tag{1}$$

where ρ stands for specific mass.

The term q_r^V is a spherically symmetric, time-dependent, volumetric heat source, and T_m is the space–time-dependent temperature of the medium.

In dimensionless form, and for constant properties, Eq. (1) is written as

$$\left[\frac{\kappa_m \langle T\rangle \lambda}{a^2}\right]\frac{\partial \hat{T}_m}{\partial \hat{t}} = \left[\frac{\kappa_m \langle T\rangle}{a^2}\right]\frac{\partial}{\hat{r}^2 \partial \hat{r}}\left(\hat{r}^2 \frac{\partial \hat{T}_m}{\partial \hat{r}}\right) + [q_0^V]\hat{q}_r^V, \qquad \hat{r} > 1 \tag{2}$$

where $\hat{t}$ is the dimensionless time $\kappa_m t/\rho_p c_p a^2$.

The dependent variable $\hat{T}_m$ is the dimensionless temperature of the medium $T_m/\langle T\rangle$, and $\langle T\rangle$ is the characteristic mean temperature level of the particle and the medium $[T_p(0) + T_m(0)]/2$. Equation (2) allows quantifying the relative importance of the unsteady and diffusive terms. If the quantity λ is of order 1, both terms are equally important for the temperature field, whereas if $\lambda << 1$, the temperature field is quasisteady. The term q_0^V is a characteristic volumetric heat source of the problem. The importance of the dimensionless volumetric source term shall be evaluated when the radiation heat transfer problem is analyzed.

The spherically symmetric problem defined by Eq. (1) can be split into two parts: One because of the unperturbed temperature filed, and the other because of the presence of the particles.[3] For a uniform, time-dependent heat source, the transient temperature behavior of the particles is then given by

$$\left(\frac{\rho_p c_p a}{3}\right)\frac{\mathrm{d}T_p}{\mathrm{d}t} = \kappa_m \left.\frac{\partial T_m(r,t)}{\partial r}\right|_{r=a} + \left(\frac{\rho_m c_m a}{3}\right)\frac{\mathrm{d}T_m(\infty, t)}{\mathrm{d}t} \tag{3}$$

where the first term on the right-hand side (RHS) is the term originated by the presence of the particles, and the second term is from a uniform volumetric heat source in the background field.

Michaelides and Feng[3] showed that the manipulation of Eqs. (1) and (3) allows describing the temperature history of an infinitely conductive particle in terms of an equation analogous to the momentum equation for the motion of a small sphere in a nonuniform flow.[4] They derived an implicit equation for the heat transfer rate at the surface of the particle by Laplace-transforming Eq. (1), with appropriate semi-infinite boundary conditions. The resulting equation describes the behavior of the temperature potential between the transient temperature of the particle and the transient conditions at infinity. In this work we concentrate on the problem of a uniform, time-dependent temperature field that is made nonuniform by the presence of the particles only. In this situation, the background or unperturbed temperature field is only a function of time. Equations (1) and (3) and the appropriate semi-infinite boundary and initial conditions are then transformed into the following implicit, ordinary, dimensionless, integro-differential equation, which is a simplification of the equation derived by Michelides and Feng for the case of uniform background temperature field:

$$\frac{\mathrm{d}\theta}{\mathrm{d}\hat{t}} = (\lambda - 1)\frac{\mathrm{d}\tilde{T}_m}{\mathrm{d}\hat{t}} - 3\theta - 3\sqrt{\frac{\lambda}{\pi}}\left[\int_0^{\hat{t}}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)\frac{1}{\sqrt{\hat{t}-\tau}}\mathrm{d}\tau + \frac{\theta(0)}{\sqrt{\hat{t}}}\right] \tag{4}$$

In Eq. (4), θ is the dimensionless temperature potential $(T_p - \tilde{T}_m)/\langle T\rangle$, and $\tilde{T}_m$ is the dimensionless, "unperturbed," time-dependent temperature of the medium at the position of the particle. Unperturbed here is used to mean in the absence of the particles. Equation (4) is valid in the limit of infinitesimal Péclet ($Pe = RePr$) and Biot (Bi) numbers. The infinitesimal Péclet number restriction exists because the convective terms were neglected a priori in Eq. (1). The small Biot number restriction exists because the temperature of the particle is assumed uniform but time dependent throughout its volume.

Equation (4) yields a trivial solution for $\lambda = 1$, which means that the temperature of the particle follows exactly the temperature fluctuations of the medium in this case. This solution is expected because the term containing the time derivative of the medium temperature is zero for $\lambda = 1$. The term containing the time derivative of the medium temperature is hereafter called the *unsteady forcing* term, because it is responsible for inducing the transient thermal behavior of the particles. The third term on the RHS of Eq. (4) is analogous to the Basset or history term in the particle momentum equation[8] and is hereafter called the *history* term.

In the following derivation it is assumed that the average distance between particles (linear separation) is much larger than the particle radius, a condition often found in many engineering applications. This assumption means that the diffusion occurs in an approximately infinite medium, or in other words, the diffusion process occurs in the proximity of the particle and is insensitive to other particles in the cloud. The presence of the other particles is felt only through the radiation heat transfer term. It should be noticed that the perturbation caused by a single particle in the temperature field of the local surrounding medium is accounted for in Eq. (4), which is the exact diffusive formulation of the problem for the case of a single particle and zero Péclet and Biot numbers.

III. Radiation Problem

Consider a dilute cloud of particles embedded within a continuous gray medium. The volume of the continuous phase is V_m, and the particle number density is N_p. The geometric mean beam length L_{mbg} is then given by[9]

$$L_{\mathrm{mbg}} = \frac{4V_m}{A_{\mathrm{sp}}} = \frac{[1 - (4\pi a^3 N_p/3)]}{N_p \pi a^2} \tag{5}$$

where A_{sp} is the surface area of the particles. With the geometric mean beam length and the absorption coefficient of the continuous phase k_{am}, the emissivity of the medium is estimated by $\epsilon_m \approx 1 - \exp(-k_{\mathrm{am}}/L_{\mathrm{mbg}})$.

The radiosity q^+ and the irradiation q^- of the particles are given by the following equations[9]:

$$q_p^+ = \epsilon_p B_p + (1 - \epsilon_p) q_p^- \tag{6}$$

$$q_p^- = \epsilon_m B_m + (1 - \epsilon_m) q_p^+ \tag{7}$$

where B_j represents the blackbody radiosity at temperature j, σT_j^4. It is implied in Eqs. (6) and (7) that the spherical layer of medium adjacent to the particle is optically thin because it does not participate in the radiation heat transfer between the particles and the medium. This assumption does not restrict significantly the result of the analysis because it is easily satisfied for a small particle and for small absorption coefficients. The diffusion radius of influence can be estimated to be of the order of 10 times the radius of the particle.[10] It is a good approximation to assume a nonparticipating layer for this radius of influence, even for large absorption coefficients, given that the radius of the particle is of the order of 1 mm or smaller. A more thorough discussion of what is meant by small absorption coefficient is given in the end of this section. Note that Eq. (6) can be applied approximately to a partially transparent particle as well as to an opaque one.

The net radiative heat flux onto the particle surface is $q_p^- - q_p^+$

$$q_{\mathrm{rad,in}} = \frac{\epsilon_m \epsilon_p (B_m - B_p)}{1 - (1 - \epsilon_m)(1 - \epsilon_p)} \tag{8}$$

Equation (8) can be linearized for the case of small temperature perturbations to yield

$$q_{\mathrm{rad,in}} = \frac{4\epsilon_m \epsilon_p \sigma \langle T\rangle^3 (T_m - T_p)}{1 - (1 - \epsilon_m)(1 - \epsilon_p)} \tag{9}$$

If a nongray medium is considered, an approximation for the incoming heat flux is given by

$$q_{\mathrm{rad,in}} = \frac{4\epsilon_{m,i} \epsilon_p \sigma \langle T\rangle^3 (T_m - T_p)}{1 - \tau_{m,i}(1 - \epsilon_p)} \tag{10}$$

where the subscript i in the properties of the medium stands for internal optical property p_i, defined as[9]

$$p_i \equiv \int_0^\infty B_\nu \frac{\partial B_\nu}{\partial T} p_\nu \, \mathrm{d}\nu / 4\sigma \langle T\rangle^3 \tag{11}$$

To close the radiation heat transfer problem, the influence of the volumetric radiation heat source in the region adjacent to the particle must be analyzed. To estimate this contribution, which is a result of the temperature potential in the near region around the particle, consider a point situated at a distance r from the center of the near particle. The portion of the mean intensity of the irradiation coming from the particle is approximately equal to[9]

$$I_{r,p} \approx \frac{\{1 - \cos[\sin^{-1}(a/r)]\} q_p^+}{2\pi}, \qquad r > a \tag{12}$$

and that of the radiation coming from the remaining solid angle is estimated by[9]

$$I_{r,m} \approx \frac{\{1 + \cos[\sin^{-1}(a/r)]\} q_p^-}{2\pi}, \qquad r > a \tag{13}$$

The volumetric heat source in the medium as a result of the radiation at location r is then

$$q_r^V \approx 4\pi k_{\mathrm{am}} (I_{r,p} + I_{r,m}) - 4k_{\mathrm{am}} B_m(r) \tag{14}$$

An upper-bound estimate for the order of magnitude of $\hat{q}_r^V$ in Eq. (2) is then

$$\hat{q}_r^V \sim \mathcal{O}\left(\frac{k_{\mathrm{am}} \sigma \langle T\rangle^4 \theta^4}{q_0^V}\right) \tag{15}$$

Comparing this upper-bound estimate with the unsteady term in Eq. (2), we see that to neglect the influence of the volumetric heat source a restriction on the absorption coefficient must be imposed. This restriction is

$$k_{\mathrm{am}} << \frac{\lambda \kappa_m}{\sigma a^2 \langle T\rangle^3} \tag{16}$$

The condition given by Eq. (16) will be further analyzed in the next section. In the derivation that follows the particles are assumed to be at the same temperature of the medium before the temperature of the latter is disturbed.

IV. Coupling Between Modes

As an analogy with linearized fluid mechanics, the second (conduction) term on the RHS of Eq. (4) represents a steady-state drag term for the particle temperature. This term will be referred as the *steady-state conduction* (SSC) term. Because the diffusion problem is linear, and the radiation problem has been linearized, the radiation effect is superimposed to the diffusion heat transfer problem. Inclusion of the radiation requires an account of Eq. (9), which, after nondimensionalization, becomes

$$\hat{Q}_{\mathrm{rad,in}} = \frac{-12a\epsilon_m \epsilon_p \sigma \langle T\rangle^3 \theta}{\kappa_m [1 - \tau_m (1 - \epsilon_p)]} \tag{17}$$

where for a gray medium $\tau_m = 1 - \epsilon_m$ and for a nongray medium both τ_m and ϵ_m are total internal properties. The diffusion–radiation equation describing the transient temperature potential for the particles is then

$$\frac{d\theta}{d\hat{t}} + H\int_0^{\hat{t}}\left(\frac{d\theta}{d\tau}\right)\frac{1}{\sqrt{\hat{t}-\tau}}\,d\tau + D\theta = (\lambda - 1)\frac{d\tilde{T}_m}{d\hat{t}} \tag{18}$$

where the following coefficients are defined to simplify the notation in the derivation that follows:

$$D = 3(1 + C_R) \tag{19}$$

$$H = 3\sqrt{\lambda/\pi} \tag{20}$$

$$U = \lambda - 1 \tag{21}$$

C_R is a defined as

$$C_R = \frac{4a\epsilon_m\epsilon_p\sigma\langle T\rangle^3}{\kappa_m[1 - \tau_m(1 - \epsilon_p)]} \tag{22}$$

This term is a measure of the contribution of the radiation mode to the temperature behavior of the particles. If $C_R << 1$, radiation effects can be neglected. However, in many engineering applications, the order of magnitude of C_R may range from 1 to much greater than 1, implying that radiation can contribute dominantly to the SSC term in Eq. (18).

The restriction on the absorption coefficient is now re-evaluated in terms of the C_R coefficient. In terms of C_R, k_{am} assumes the simple form

$$k_{\mathrm{am}} << 4\lambda/C_R a \tag{23}$$

As an example of practical interest, if C_R is equal to 1 and λ/a is equal to 10 m^{-1}, k_{am} is conditioned by Eq. (23) to be much smaller than 40 m^{-1}. The restriction in this particular situation is not practically binding, being satisfied even for many (not too sooty) flame environments.

For small particles ($a \lesssim 1$ mm), the radiation contribution becomes only relevant at high-temperature levels ($T \gtrsim 1000$ K), which allows linearizing the radiation term [Eq. (9)]. If condition (23) is also satisfied (as it is in many engineering problems), the radiation contribution can be correctly described by modifying the steady-state conduction term to include the radiation coefficient C_R. The practical result of this theoretical analysis is that the importance of the radiation contribution for small particles can be easily assessed, and given that condition (23) is met, the solution of the coupled problem can be found in a general way.

V. Solution of the Radiation-Diffusion Equation

Equation (18) can be solved numerically for a given unperturbed time-dependent temperature field of the medium $\tilde{T}_m(\hat{t})$. The numerical procedure has to be iterative and, thus, time consuming because of the implicit nature of the equation. Moreover, solving numerically gives little insight to the contribution of the different heat transfer mechanisms in this problem. If we resist the temptation of solving Eq. (18) numerically, we see that this equation can be solved analytically. First, the integral term in Eq. (18) is recognized as $\sqrt{\pi}$ times the Riemann–Liouville–Weyl half-derivative of θ. The Riemann–Liouville–Weyl fractional derivative of order n of $f(\hat{t})$ is defined as[11]

$$\frac{d^n f(\hat{t})}{d\hat{t}^n} = \begin{cases} 1/\Gamma(-n)\displaystyle\int_{-\infty}^{\hat{t}} (\hat{t}-\sigma)^{-n-1} f(\sigma)\,d\sigma, & n < 0 \\ \dfrac{1}{\Gamma(m-n)}\dfrac{d^n}{d\hat{t}^n}\left(\displaystyle\int_{-\infty}^{\hat{t}} (\hat{t}-\sigma)^{m-n-1} f(\sigma)\,d\sigma\right), & n \geq 0 \end{cases} \tag{24}$$

where $\Gamma(z)$ is the gamma (generalized factorial) function of z, and $m - 1 \leq n < m$, $m = 1, 2, 3, \ldots$.

We define the following conjugate linear operators $\Psi^{\pm}$ that are composed of three terms, one of them containing a Riemann–Liouville–Weyl half-derivative:

$$\Psi^{\pm} = \frac{d}{d\hat{t}} \pm H\sqrt{\pi}\frac{d^{1/2}}{d\hat{t}^{1/2}} + D \tag{25}$$

The left-hand side (LHS) of Eq. (18) is then simply $\Psi^+[\theta(\hat{t})]$. The initial step in the solution procedure outlined here consists of applying the fractional-differential operator Ψ^- to Eq. (18). The objective of this procedure is to stretch the half-derivative associated with the history term in that equation. The following application of Ψ^- to Eq. (18)

$$\Psi^-\{\Psi^+[\theta(\hat{t})]\} = U\Psi^-\left\{\frac{d\tilde{T}_m}{d\hat{t}}\right\} \tag{26}$$

results in

$$\frac{d^2\theta}{d\hat{t}^2} + (2D - H^2\pi)\frac{d\theta}{d\hat{t}} + D^2\theta = -U\frac{d^2\tilde{T}_m}{d\hat{t}^2} - DU\frac{d\tilde{T}_m}{d\hat{t}} + HU\left(\frac{1}{\sqrt{\hat{t}}}\left.\frac{d\tilde{T}_m}{d\hat{t}}\right|_{\hat{t}=0} + \int_0^{\hat{t}}\frac{d^2\tilde{T}_m}{d\tau^2}\frac{d\tau}{\sqrt{\hat{t}-\tau}}\right) \tag{27}$$

Transformation of integro-differential equations such as Eq. (4) into higher-order, ordinary differential equations (ODEs) can also be done by Laplace transformation of the original equation and its initial conditions. Konopliv[12] first proposed this procedure for the case of a constant forcing term. Michaelides[13] extended Konopliv's method for the analogous equation of motion with time-dependent forcing terms. Neither Konopliv nor Michaelides solved the resulting ODE by analytical methods. The Laplace transformation of integro-differential equations involves many intermediary algebraic steps in the Laplace variable that are avoided by using the linear operators defined in Eq. (25). For a more detailed discussion of the analogies between the momentum and heat transfer equations for small particles in the limit of zero Reynolds and Peclét numbers the reader should consult Ref. 14.

Equation (27) is valid for initial temperature equilibrium between the phases [initial condition (32)], and for a generic piecewise continuous temperature fluctuation of the background field. This equation can now be solved exactly. To solve Eq. (27) the nature of the solution has to be analyzed. There is a critical value of λ for which the functional form of the solution changes character. This value is $\lambda_c = 4(1 + C_R)/3$. Three possible cases exist: $\lambda > \lambda_c$, $\lambda = \lambda_c$, and $\lambda < \lambda_c$. The first two cases are of limited practical interest because they imply a suspension of particles with infinite conductivity but with a density smaller than that of the surrounding medium, because the value of the specific heat capacities of solids, liquids, and gases are of the same order of magnitude. Because of this implication, and because the solution for the first two cases is actually simpler than the solution for the third case, they will not be considered here.

The solution of Eq. (27) for $\lambda < \lambda_c$ is then found using variation of parameters to yield

$$\theta(\hat{t}) = \frac{e^{-\alpha\hat{t}}}{\beta}\left[+U\left.\frac{d\tilde{T}_m}{d\hat{t}}\right|_{\hat{t}=0}\sin(\beta\hat{t}) - \sin(\beta\hat{t})\int_0^{\hat{t}} e^{\alpha\tau}\cos(\beta\tau)rhs(\tau)\,d\tau + \cos(\beta\hat{t})\int_0^{\hat{t}} e^{\alpha\tau}\sin(\beta\tau)rhs(\tau)\,d\tau\right] \tag{28}$$

where $rhs(\hat{t})$ is

$$rhs(\hat{t}) = HU\left(\int_0^{\hat{t}} \frac{d^2\tilde{T}_m\, d\tau}{d\tau^2\sqrt{\hat{t}-\tau}} + \frac{1}{\sqrt{\hat{t}}}\frac{d\tilde{T}_m}{d\hat{t}}\bigg|_{\hat{t}=0}\right) - U\frac{d^2\tilde{T}_m}{d\hat{t}^2} - DU\frac{d\tilde{T}_m}{d\hat{t}} \tag{29}$$

and the coefficients α and β are defined as

$$\alpha = D - (H^2\pi/2) \tag{30}$$

$$\beta = \sqrt{|\Delta|/2} \tag{31}$$

To obtain Eq. (28), the following initial conditions were used:

$$\theta(0) = 0 \tag{32}$$

$$\frac{d\theta}{d\hat{t}}\bigg|_{\hat{t}=0} = U\frac{d\tilde{T}_m}{d\hat{t}}\bigg|_{\hat{t}=0} \tag{33}$$

The first initial condition is due to the original assumption of initial thermodynamic equilibrium between the particles and the surrounding medium. The second condition is derived directly from Eq. (18).

Equation (28) is the general solution for the dimensionless temperature potential. By adding to it the value of the unperturbed temperature of the medium $\tilde{T}_m(\hat{t})$, the dimensionless temperature of the particle is found. The next two sections treat specific cases of $\tilde{T}_m(\hat{t})$, namely, the case in which the temperature of the medium increases linearly in time and the case in which the temperature of the medium varies harmonically in time.

VI. Temperature of the Medium Increasing Linearly with Time

Consider the case when the dimensionless temperature of the medium is $\tilde{T}_m(\hat{t}) = 1 + \varepsilon\hat{t}$. The term $rhs(t)$ is then simply

$$rhs_L(\hat{t}) = -DU\varepsilon + \frac{HU\varepsilon}{\sqrt{\hat{t}}} \tag{34}$$

The time-dependent behavior of θ is found through Eq. (28) to be

$$\theta_L(\hat{t}) = \frac{U\varepsilon}{D} + \frac{e^{-\alpha\hat{t}}}{\beta}\Bigg\{+U\varepsilon\sin(\beta\hat{t}) - U\varepsilon\left[\frac{\beta\cos(\beta\hat{t}) + \alpha\sin(\beta\hat{t})}{D}\right] - HU\varepsilon\sin(\beta\hat{t})\int_0^{\hat{t}}\frac{e^{\alpha\tau}}{\sqrt{\tau}}\cos(\beta\tau)\,d\tau + HU\varepsilon\cos(\beta\hat{t})\int_0^{\hat{t}}\frac{e^{\alpha\tau}}{\sqrt{\tau}}\sin(\beta\tau)\,d\tau\Bigg\} \tag{35}$$

Unfortunately, the two definite integrals appearing in Eq. (35) cannot be solved analytically. The numerical integration of these two integrals is, however, a simple task.

To study the effect of the history term, let the value of H be identically zero. Equation (18) subjected to initial condition (32) is easily solved analytically to give

$$\theta_L(\hat{t}) = \frac{U\varepsilon}{D}(1 - e^{-D\hat{t}}) \tag{36}$$

When history term effects are not taken into account, it is easy to quantify the importance of the radiation contribution. When C_R is equal to 1, the exponential decay rate is twice as large as the one found neglecting radiation effects [see Eq. (19)].

Figure 1 shows the behavior of the normalized temperature of the particles as a function of the dimensionless time $\hat{t}$ for two different values of λ (0.001 and 0.1), and for $C_R = 0$, meaning in the absence of radiation effects. The case $C_R = 0$ is relevant for dilute liquid–solid suspensions because most liquids are opaque to radiation heat transfer. In these suspensions the analysis presented in this work is only limited to low Péclet and Biot numbers. In practical systems these limitations are satisfied when the solid phase is constituted of highly conductive small particles.

The thick curves in Fig. 1 correspond to the full solution (35), and the thin curves correspond to the solution neglecting the history term (36). The normalized, unperturbed temperature of the medium is also plotted for comparison. As can be inferred from Fig. 1, the history term effect becomes important as the value of λ deviates from zero. The enhanced heat transfer given by the additional contribution of the history term for initial times is evident by the higher particle temperatures in the full solution when compared to the solution neglecting the history term.

Figures 2 and 3 are analogous to Fig. 1, but for values of C_R equal to 1 and 10, respectively. The radiation contribution can be evaluated by comparing Figs. 1, 2, and 3. The terminal temperature potential is $U\varepsilon/D$, and it is smaller for larger radiation effects.

Because the linearized radiation term acts as an enhanced SSC term, the curves for different values of λ approach each other for larger values of C_R. Radiation has the effect of diminishing the differences between curves for different λ, and also of reducing the history term effect. For C_R equal to 10 (Fig. 3), the history term effects are negligible because the SSC term becomes dominant.

Increasing radiation also makes the particles reach the normalized temperature potential ($U\varepsilon/D$) in a shorter dimensionless time $\hat{t}$. This means that when radiation effects become relevant ($C_R >> 0$), the transient to reach the terminal temperature potential is significantly reduced. For $C_R = 10$, solution (36) reaches the terminal temperature potential in a fraction of the characteristic time $\hat{t}$. Although the history term contribution asymptotes to zero for extended times, Fig. 2 shows that for moderate radiation effects, and for a value of λ equal to 0.1,

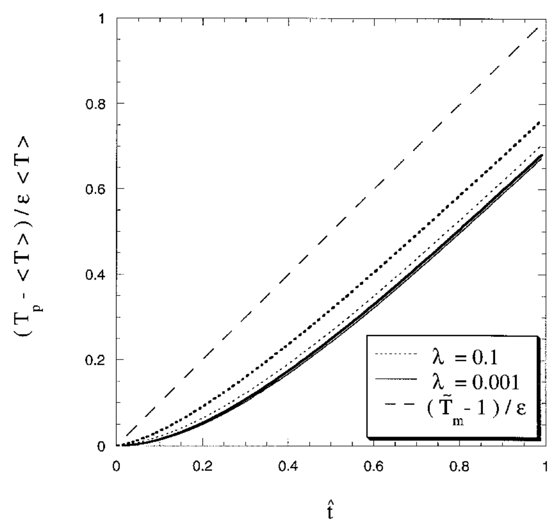

Fig. 1 Normalized temperature of the particles as a function of time $\hat{t}$ [Eq. (35)] for two different values of λ (0.001, 0.1) and for the case $\tilde{T}_m(\hat{t}) = 1 + \varepsilon\hat{t}$. The thin curves were calculated using Eq. (36), where the history term effect is neglected. The thick curves correspond to the full solution (35). $C_R = 0$ for all curves.

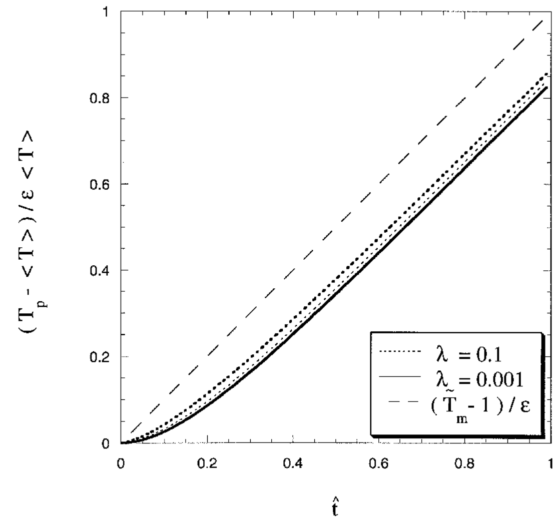

Fig. 2 As in Fig. 1, but for $C_R = 1$.

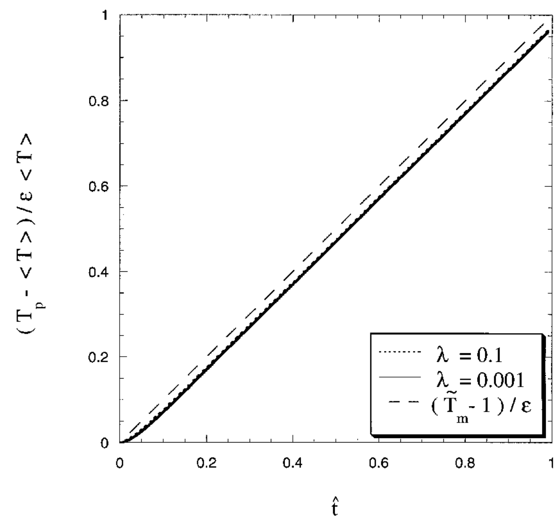

Fig. 3 As in Figs. 1 and 2, but for $C_R = 10$.

solution (35) does not reach the terminal temperature potential in one characteristic time $\hat{t}$. This is because the history term contribution decreases very slowly with $\hat{t}^{-1/2}$.

VII. Temperature of the Medium Varying Harmonically with Time

Consider now the case when the temperature of the medium varies harmonically as $\tilde{T}_m(\hat{t}) = 1 + \xi \sin(\omega \hat{t})$. The term $rhs(\hat{t})$ in this case is

$$rhs_H(\hat{t}) = \xi\omega U \left[+\omega \sin(\omega\hat{t}) - D\cos(\omega\hat{t}) + \frac{H}{\sqrt{\hat{t}}} - \omega H \int_0^{\hat{t}} \frac{\sin(\omega\tau)\,d\tau}{\sqrt{\hat{t}-\tau}} \right] \tag{37}$$

The dimensionless temperature potential is then given by

$$\theta_H(\hat{t}) = \frac{e^{-\alpha\hat{t}}}{\beta} \left[+U\xi\omega \sin(\beta\hat{t}) - \sin(\beta\hat{t}) \int_0^{\hat{t}} e^{\alpha\tau} \cos(\beta\tau) rhs_H(\tau)\,d\tau + \cos(\beta\hat{t}) \int_0^{\hat{t}} e^{\alpha\tau} \sin(\beta\tau) rhs_H(\tau)\,d\tau \right] \tag{38}$$

For small values of the dimensionless frequency of perturbation ω, the first and fourth terms in the RHS of Eq. (37) become negligible. In other words, for slow oscillations of the medium temperature, the value of the integrals in Eq. (38) is controlled by the SSC term and by the history term that is not associated with the memory integral [the definite integral in Eq. (37) is called hereafter the *memory integral*].

For small-frequency oscillations and large values of C_R, $rhs_H(\hat{t})$ is controlled by the SSC term only, because the value of λ is at most of the order of 1. For high values of ω, the memory integral term and the first term in the RHS of Eq. (37) dominate the behavior of $rhs_H(\hat{t})$. In many cases of practical interest, the density of the particles is much larger than the density of the medium, and the specific heat capacities of the two media are of the same order, making the value of λ very small. For very small values of λ, and at high-frequencies ω, the first term in the RHS of Eq. (37) is dominant over the memory integral term for all times except the very initial ones. For the same small values of λ, but at low-frequencies ω, the SSC term dominates. In these particular situations the history term contribution can be neglected, and a solution to Eq. (18) neglecting the history term contributions is

$$\theta(\hat{t}) = \frac{U\xi\omega D}{D^2 + \omega^2} \left[\cos(\omega\hat{t}) - \frac{\omega}{D}\sin(\omega\hat{t}) - e^{-D\hat{t}} \right] \tag{39}$$

Equation (39) shows that when C_R is large and when the frequency ω is small, the temperature potential is out of phase with respect to the medium perturbation. In this case, the cosine term is dominant in Eq. (39). When ω is large and radiation is not strong, the differential response asymptotically approaches an oscillatory behavior that is in phase with the perturbation, but with opposite signs (note that U is negative for λ smaller than 1). The temperature of the particles is thus unperturbed for long times when ω is large and D is small. For the case when λ is not very small, solution (38) has to be evaluated either analytically or numerically. From Eqs. (37) and (38) it is seen that all definite integrals appearing in the expression for $\theta_H(\hat{t})$ can be evaluated analytically, except for the ones that originate directly from the terms containing the factor H in Eq. (37). The integrals that originate from the first two terms in Eq. (37) can be solved exactly with the help of the following primitives:

$$\begin{aligned} I_1 &= \int e^{\alpha x} \cos(\beta x)\sin(\omega x)\,dx \\ &= \frac{e^{\alpha x}}{2(\alpha^4 + 2\alpha^2\beta^2 + \beta^4 + 2\alpha^2\omega^2 - 2\omega^2\beta^2 + \omega^4)} \\ &\times \{(\alpha^2\beta + \beta^3 - \alpha^2\omega + \beta^2\omega - \beta\omega^2 - \omega^3)\cos[(\beta - \omega)x] \\ &+ (\omega^2\beta - \beta^3 - \alpha^2\beta + \beta^2\omega - \beta\omega^2 - \omega^3)\cos[(\beta + \omega)x] \\ &- (\alpha\omega^2 + \alpha^3 + \alpha\beta^2 + 2\alpha\beta\omega)\sin[(\beta - \omega)x] \\ &+ (\alpha\omega^2 + \alpha^3 + \alpha\beta^2 - 2\alpha\beta\omega)\sin[(\beta + \omega)x]\} \end{aligned} \tag{40}$$

$$\begin{aligned} I_2 &= \int e^{\alpha x} \sin(\beta x)\sin(\omega x)\,dx \\ &= \frac{e^{\alpha x}}{2(\alpha^4 + 2\alpha^2\beta^2 + \beta^4 + 2\alpha^2\omega^2 - 2\omega\beta^2 + \omega^4)} \\ &\times \{(\alpha^3 + \beta^2\alpha + 2\alpha\beta\omega + \alpha\omega^2)\cos[(\beta - \omega)x] \\ &+ (2\alpha\beta\omega - \alpha^3 - \alpha\beta^2 - \omega^2\alpha)\cos[(\beta + \omega)x] \\ &+ (\alpha^2\beta + \beta^3 - \alpha^2\omega + \beta^2\omega - \beta\omega^2 - \omega^3)\sin[(\beta - \omega)x] \\ &- (\alpha^2\beta + \beta^3 + \alpha^2\omega - \beta^2\omega - \beta\omega^2 + \omega^3)\sin[(\beta + \omega)x]\} \end{aligned} \tag{41}$$

$$I_3 = \int e^{\alpha x}\cos(\beta x)\cos(\omega x)\,\mathrm{d}x$$
$$= \frac{e^{\alpha x}}{2(\alpha^4 + 2\alpha^2\beta^2 + \beta^4 + 2\alpha^2\omega^2 - 2\omega^2\beta^2 + \omega^4)}$$
$$\times (\alpha^3 + \alpha\beta^2 + 2\alpha\beta\omega + \alpha\omega^2)\cos[(\beta - \omega)x]$$
$$+ (\alpha^3 + \alpha\beta^2 - 2\alpha\beta\omega + \alpha\omega^2)\cos[(\beta + \omega)x]$$
$$+ (\alpha^2\beta + \beta^3 - \alpha^2\omega + \beta^2\omega - \beta\omega^2 - \omega^3)\sin[(\beta - \omega)x]$$
$$+ (\alpha^2\beta + \beta^3 + \alpha^2\omega - \beta^2\omega - \beta\omega^2 + \omega^3)\sin[(\beta + \omega)x]\} \tag{42}$$

The fourth and last primitive (I_4) can be derived by switching β for ω, and vice versa in Eq. (40). The temperature potential is then given by

$$\theta_H(\hat{t}) = \frac{\xi\omega U e^{-\alpha\hat{t}}}{\beta}\omega\sin(\omega\hat{t}) + \frac{\xi\omega U e^{-\alpha\hat{t}}}{\beta}\cos(\omega\hat{t})$$
$$\times \left\{\omega I_2|_0^{\hat{t}} - D I_4|_0^{\hat{t}} + H\int_0^{\hat{t}}\frac{e^{\alpha\tau}\sin(\beta\tau)}{\sqrt{\tau}}\mathrm{d}\tau\right.$$
$$\left. - \omega H\int_0^{\hat{t}}\left[e^{\alpha\tau}\sin(\beta\tau)\int_0^{\tau}\frac{\sin(\omega\sigma)}{\sqrt{\tau - \sigma}}\mathrm{d}\sigma\right]\mathrm{d}\tau\right\}$$
$$- \frac{\xi\omega U e^{-\alpha\hat{t}}}{\beta}\sin(\omega\hat{t}) \times \left\{\omega I_1|_0^{\hat{t}} - D I_3|_0^{\hat{t}} + H\int_0^{\hat{t}}\frac{e^{\alpha\tau}\sin(\beta\tau)}{\sqrt{\tau}}\mathrm{d}\tau\right.$$
$$\left. - \omega H\int_0^{\hat{t}}\left[e^{\alpha\tau}\cos(\beta\tau)\int_0^{\tau}\frac{\sin(\omega\sigma)}{\sqrt{\tau - \sigma}}\mathrm{d}\sigma\right]\mathrm{d}\tau\right\} \tag{43}$$

The analytical evaluation of the integrable terms in Eq. (38) is not very practical because of the number of terms involved in Eq. (43), making the numerical evaluation of solution (38) a better choice to study individual cases. For any particular choice of ω and λ, Eq. (38) can be readily evaluated. The substantial advantage of evaluating solution (38) numerically as opposed to solving Eq. (27) numerically should not be underestimated. Equation (27) is subjected to numerical stiffness for some values of λ, requiring a numerical solver much more sophisticated than simple integration algorithms. Furthermore, the contribution of the individual terms in solution (38) is much more explicit than in Eq. (27), allowing a better understanding of the problem even before any solution is reached.

Figure 4 shows the normalized temperature of the particles when radiation is not considered ($C_R = 0$) and for a dimensionless angular frequency of the medium perturbation ω equal to 10. The curves corresponding to solutions (38) and (39) were evaluated numerically through the 1/3 Simpson rule algorithm for two different values of λ. The harmonic, unperturbed temperature is also shown for comparison. As in the linear perturbation case, the no-radiation case is relevant for liquid–solid suspensions and for low-temperature gas–solid suspensions. The particle temperature lags the harmonic perturbation of the medium, and for zero radiation effects and for a value of $\omega = 10$ the amplitude of the particle temperature response can be as low as 40% of the forcing amplitude ξ. As expected, the amplitude of the response increases as the value of λ approaches 1. Because both the history-term contribution and the radiation term act as mechanisms of conduction enhancement, the thick curves corresponding to the full solution respond faster to the thermal perturbations of the medium, always lagging less than the simplified solution given by Eq. (39).

Figures 5 and 6 are analogous to Fig. 4, but correspond to values of C_R equal to 1 and 10, respectively. Figure 5 shows that for moderate radiation effects the history effects are negligible if the value of λ approaches zero, but become important for higher values of λ. At higher values of C_R Fig. 6

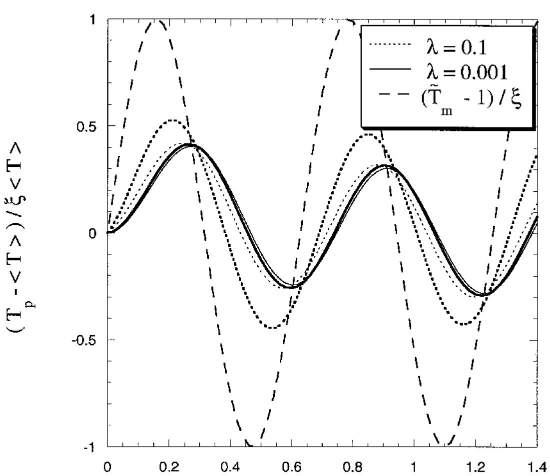

Fig. 4 Normalized temperature of the particles as a function of time $\hat{t}$ for two different values of λ (0.001, 0.1) and for the case $\tilde{T}_m(\hat{t}) = 1 + \xi\sin(\omega\hat{t})$. The thin curves were calculated using Eq. (39), where the history term effect is neglected. The thick curves correspond to the full solution (38). $C_R = 0$ and $\omega = 10$ for all curves.

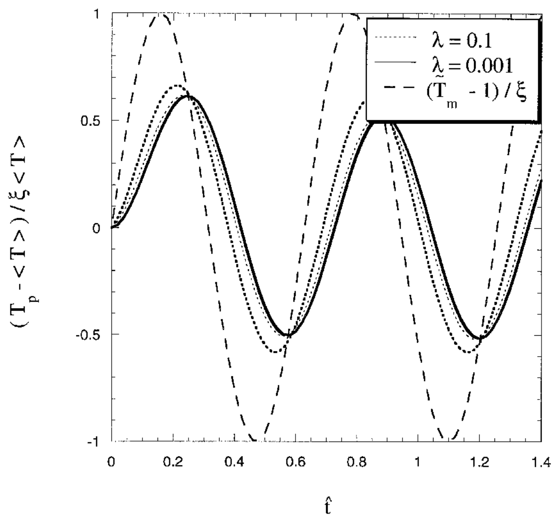

Fig. 5 As in Fig. 4, but for $C_R = 1$.

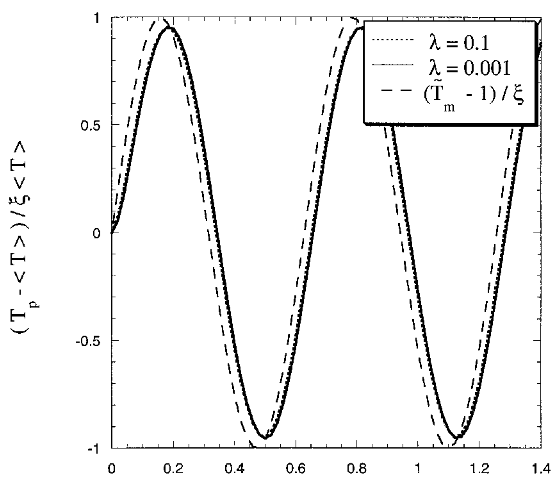

Fig. 6 As in Figs. 4 and 5, but for $C_R = 10$.

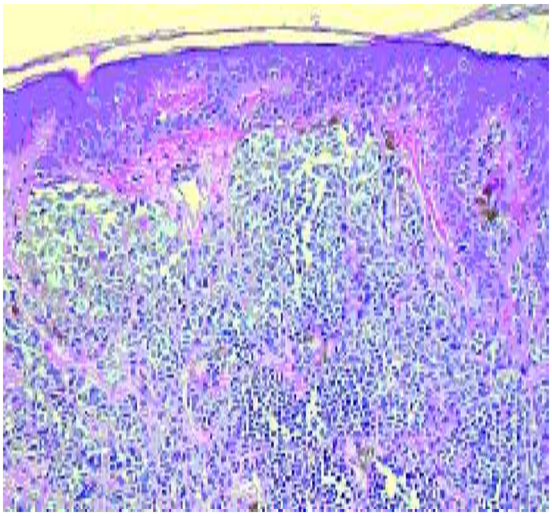

Fig. 7 Normalized temperature of the particles as a function of time $\hat{t}$ for two different values of λ (0.001, 0.1), and for the case $\tilde{T}_m(\hat{t}) = 1 + \xi \sin(\omega \hat{t})$. The thin curves were calculated using Eq. (39), where the history term effect is neglected. The thick curves correspond to the full solution [Eq. (38)]. $C_R = 0$ and $\omega = 1$ for all curves.

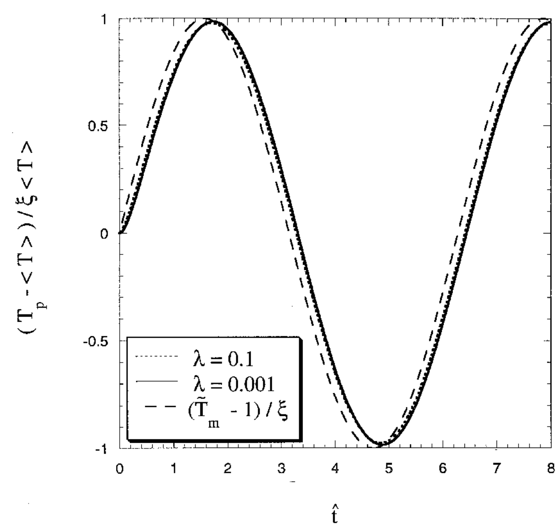

Fig. 8 As in Fig. 7, but for $C_R = 1$.

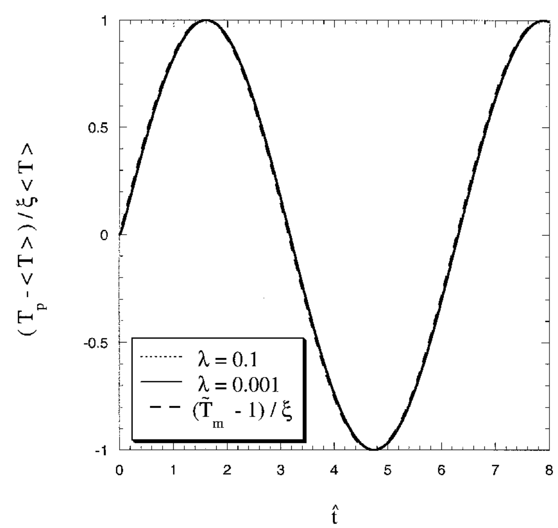

Fig. 9 As in Figs. 7 and 8, but for $C_R = 10$.

shows that the temperature of the particles approaches the unperturbed temperature of the medium.

If the forcing frequency ω is reduced the particles have time to respond to the perturbation of the medium temperature even when radiation effects are not important. Figure 7 shows that the curves for λ equal to 0.001 are undistinguishable, and that the error of neglecting the history term when the forcing frequency ω is of order 1 is small. The lagging caused by a different thermal inertia is still significant, but the use of solution (39) becomes justifiable for values of ω smaller than 1, even when radiation is not significant and λ approaches 1. Figures 8 and 9 show plots analogous to those in Fig. 7, but for higher values of C_R. For $C_R = 1$ there is still a significant lagging between the phases. This lagging is negligible for $C_R = 10$.

VIII. Concluding Remarks

A theoretical description of the diffusive and radiative heat transfer problem of a homogeneous dispersion of highly conductive particles is presented. The particles and the medium surrounding them are considered to be initially in thermodynamic equilibrium, and the medium is considered to be weakly participating. The first condition (zero initial temperature potential) was chosen to simplify the derivation, but is not a necessary condition for the success of the methods applied in this work. The second condition (weakly participating medium) restricts the value of the mean absorption coefficient for which the theory applies. The range of valid values for the mean absorption coefficient is discussed in Secs. III and IV. The valid range is shown to not be very restrictive, allowing the application of the present theory to relevant industrial processes. Approximations for gray and nongray surrounding media are discussed.

The general solution for the integro-differential equations that describes the diffusion and the radiation heat transfer between the phases is derived. To illustrate specific applications of the general analytical solution of the problem described in this work, the effect of linear and harmonic thermal perturbations on the temperature potential between the surrounding medium and the particles is analyzed. It is shown that the history term contribution is small whenever the heat capacity ratio λ approaches zero. The temperature response of the particle approaches the quasisteady behavior in this case for linear and low-frequency harmonic perturbations. The quasisteady behavior is also approached when the radiation mode becomes dominant over the unsteady diffusion heat transfer mode. This last result is independent of the value of λ, because the temperature behavior of particles with different λ approach each other and the quasisteady behavior. The situations for which neglecting the history term contribution leads to inaccuracies are examined in both linear and harmonic scenarios.

Most gas–solid suspensions of practical interest are characterized by small values of λ, on the order of 0.001. In these suspensions the influence of the history term is negligible even at low temperatures. For the case of pulverized or sprayed fuel flames, the combined effects of radiation heat transfer and the characteristically small values of λ make the small particles behave quasisteadily. The situation is different for liquid–solid suspensions, where values of λ on the order of 0.1 are typical, and because radiation is usually not relevant, the deviation from the quasisteady behavior may be appreciable, particularly at higher-frequency perturbations.

Acknowledgments

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References

[1]Clift, R., Grace, J. R., and Weber, M. E., *Bubbles, Drops and Particles*, Academic, New York, 1978.

[2]Gilber, H., and Angelino, H., ‘‘Tranferts de Matière Entre une Sphère Soumise à des Vibrations et un Liquide en Mouvement,’’ *International Journal of Heat and Mass Transfer*, Vol. 17, No. 2, 1974, pp. 625–632.

[3]Michaelides, E. E., and Feng, Z. G., ‘‘Heat Transfer from a Sphere in a Non-Uniform Temperature and Velocity Field,’’ *International Journal Heat and Mass Transfer*, Vol. 37, No. 14, 1994, pp. 2069–2076.

[4]Duvvur, A., Chiang, C. H., and Sirignano, W. A., ‘‘Oscillatory Fuel Droplet Vaporization: Driving Mechanism for Combustion Instability,’’ *Journal of Propulsion and Power*, Vol. 12, No. 2, 1996, pp. 358–365.

[5]Delplanque, J.-P., and Sirignano, W. A., ‘‘Numerical Study of the Transient Vaporization on an Oxygen Droplet at Sub- and Super-Critical Conditions,’’ *International Journal of Heat and Mass Transfer*, Vol. 36, No. 2, 1993, pp. 303–314.

[6]Lage, P. L. C., and Rangel, R. H., 1993, ‘‘Single Droplet Vaporization Including Thermal Radiation Absorption,’’ *Journal of Thermophysics and Heat Transfer*, Vol. 7, No. 3, 1993, pp. 502–509.

[7]Johns, W. R., and Lawn, S. J., ‘‘Unsteady-State Transfer Between a Sphere and a Surrounding Stationary Medium with Application to Arrays of Spheres,’’ *International Journal of Heat and Mass Transfer*, Vol. 28, No. 5, 1985, pp. 1047–1053.

[8]Maxey, M. R., and Riley, J. J., ‘‘Equation of Motion for a Small Rigid Sphere in a Nonuniform Flow,’’ *Physics of Fluids*, Vol. 26, No. 4, 1983, pp. 883–889.

[9]Edwards, D. K., *Radiation Heat Transfer Notes*, Hemisphere, New York, 1981.

[10]White, F. M., *Viscous Fluid Flow*, McGraw–Hill, New York, 1991.

[11]Zayed, A. I., *Handbook of Function and Generalized Function Transformations*, CRC Press, Boca Raton, FL, 1996.

[12]Konopliv, N., ‘‘Gravitationally Induced Acceleration of Spheres in Creeping Flow—A Heat Transfer Analogy,’’ *AIChE Journal*, Vol. 17, No. 6, 1971, pp. 1502, 1503.

[13]Michaelides, E. E., ‘‘A Novel Way of Computing the Basset Term in Unsteady Multiphase Flow Computations,’’ *Physics of Fluids*, Vol. 44, No. 7, 1992, pp. 1579–1582.

[14]Michaelides, E. E., and Feng, Z. G., ‘‘Analogies Between the Transient Momentum and Energy Equations of Particles,’’ *Progress in Energy and Combustion Science*, Vol. 22, No. 2, 1996, pp. 147–162.